Reg. No.

B.E./ B.Tech (Full Time) DEGREE END SEMESTER EXAMINATION, APRIL / MAY 2024

COMPUTER SCIENCE AND ENGINEERING

SECOND SEMESTER

MA3252 - DISCRETE MATHEMATICS

(Regulation: 2023)

Time: 3 hours **Answer to all the Questions** **Maximum Marks: 100**

PART- A (10x2=20Marks)

Q. No.	Questions	Marks	CO	BL
1	Find the truth table for $P \rightarrow (P \vee Q)$.	**2**	1	1
2	Prove that " If n^2 is an odd integer, then n is also an odd integer".	**2**	1	1
3	State pigeon hole principle.	**2**	2	2
4	How many bit strings of length ten contain (i) exactly four 1's, (ii) at least four 1's?	**2**	2	2
5	How many edges are there in a graph with 10 vertices each of degree 5?	**2**	3	1
6	Find the value(s) of n so that the complete graph K_n is bipartite?	**2**	3	2
7	Show that every cyclic group must be an abelian group.	**2**	4	1
8	Give an example for an integral domain which is not a field.	**2**	4	1
9	Find the Hasse diagram of the set $X = \{1,2,3,4,6,8,12,24\}$ with division as the partial ordered relation.	**2**	5	1
10	Prove or disprove the statement: Every distributive lattice must be complemented lattice.	**2**	5	1

PART- B (5x13 = 65 Marks)

Q. No.		Questions	Marks	CO	BL
11 (a)	(i)	Obtain the principal disjunctive normal form of $(P \rightarrow \neg R) \wedge (Q \rightarrow R)$ by using equivalences.	**5**	1	3
	(ii)	Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$.	**8**	1	3
		OR			
11 (b)	(i)	Show that $((P \vee Q) \wedge 7(7P \wedge (7Q \vee 7R))) \vee (7P \wedge 7Q) \vee (7P \wedge 7R)$ is a tautology by without using truth table.	**5**	1	3
	(ii)	Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q.	**8**	1	3

12 (a)	(i)	In a room there are six men and five women. Find the number of ways four persons can be selected from that room if (1) they can be any sex, (2) two must be men and two must be women, (3) they must all are of the same sex, (4) they all of them are men.	**5**	2	4
	(ii)	Using mathematical induction show that $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.	**8**	2	4
		OR			
12 (b)	(i)	When $1 \le n \le 500$, find the number of n's which are not divisible by 2 and 3 .	**5**	2	4
	(ii)	Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2^n$ with initial conditions $a_0 = 4$ and $a_1 = 12$, for $n \ge 2$.	**8**	2	4
13 (a)	(i)	Prove or disprove the statement: The complement of a disconnected graph is always connected.	**5**	3	3
	(ii)	If G is a connected simple graph with $n \ge 3$ vertices such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that the graph G is a Hamiltonian graph.	**8**	3	3
		OR			
13 (b)	(i)	Prove that the number of odd degree vertices in any graph is even.	**5**	3	3
	(ii)	For what values of n, a graph with n vertices can be a self complementary? Justify your answer. By applying your answer, find the values of n so that each of the following graphs is self complementary? (i) Path P_n (ii) Cycle C_n	**8**	3	3
14 (a)	(i)	In a group $\langle G, * \rangle$, if the inverse of every element is itself then prove that the group $\langle G, * \rangle$ must be abelian.	**5**	4	4
	(ii)	Prove that every subgroup of a cyclic group is cyclic.	**8**	4	4
		OR			
14 (b)	(i)	Prove that the group $\langle G, * \rangle$ is abelian if and only if $(a * b)^2 = a^2 * b^2$ for all $a, b \in G$.	**5**	4	4
	(ii)	Let $f : G \to H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G.	**8**	4	4

15 (a)	(i)	Show that every distributive lattice is modular. Justify the converse of the statement.	**5**	5	3
	(ii)	Show that following in a complemented distributive lattice. $a \le b \quad \Leftrightarrow \quad a \wedge \bar{b} = 0 \quad \Leftrightarrow \quad \bar{a} \vee b = 1 \quad \Leftrightarrow \bar{b} \le \bar{a}$. Where $\bar{a}$ denotes the complement of a.	**8**	5	3
		OR			
15 (b)	(i)	Show that $\overline{a \vee b} = \bar{a} \wedge \bar{b}$ for all a, b in a Boolean Algebra.	**5**	5	3
	(ii)	Show that every chain is a distributive lattice.	**8**	5	3

PART- C (1x 15=15Marks)

(Q.No.16 is compulsory)

Q. No.		Questions	Marks	CO	BL
16.	(i)	Using mathematical logic, show that the following set of premises are inconsistent: (1) If Babu misses many classes due to illness, then he fails in high school. (2) If Babu fails in high school, then he is uneducated. (3) If Babu reads a lot of books, then he is not uneducated. (4) Babu misses many classes due to illness and he reads a lot of books.	**8**	1	6
	(ii)	Draw the graphs represented by the following adjacency matrix and exhibit an isomorphism between them or provide a rigorous argument that none exists. $\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$	**7**	3	5

*** ALL THE BEST***